1) $A=\begin{bmatrix}1&0\\1&1\end{bmatrix}$ matrisi veriliyor. $A^2-2A+3I_2$ matrisini hesaplayınız. (15 Puan)

$$A^2-2A+3I_2=\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}1&0\\1&0\end{bmatrix}-2\begin{bmatrix}1&0\\1&1\end{bmatrix}+3\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}1&0\\2&1\end{bmatrix}-\begin{bmatrix}2&0\\2&2\end{bmatrix}+\begin{bmatrix}3&0\\0&3\end{bmatrix}=\underline{\begin{bmatrix}2&0\\0&2\end{bmatrix}}$$

2) $z=f(x,y)$ olmak üzere $xz+x^2y^2z+z^2=1$ denklemi ile verilen fonksiyonu için $z_x+z_y=?$ (15 puan)

$$\partial_x(xz+x^2y^2z+z^2)=\partial_x(1) \Leftrightarrow z+xz_x+2xy^2z+x^2y^2z_x+2zz_x=0$$

$$\Rightarrow z_x(x+x^2y^2+2x)=-z-2xy^2z \Rightarrow z_x=-\frac{z+2xy^2z}{x+x^2y^2+2z} \quad \dots (1)$$

$$\partial_y(xz+x^2y^2z+z^2)=\partial_x(1) \Rightarrow xz_y+2yx^2z+x^2y^2z_y+2zz_y=0 \quad \dots (2)$$

$$\Rightarrow z_y(x+x^2y^2+2z)=-2yx^2z \Rightarrow z_y=-\frac{2yx^2z}{x+x^2y^2+2z}$$

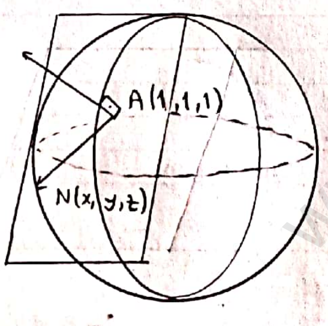
$$\overset{(1)(2)}{\Longrightarrow} z_x+z_y=-\frac{z+2xy^2z}{x+x^2y^2+2z}-\frac{2yx^2z}{x+x^2y^2+2z}=-\frac{z+2xy^2z+2yx^2z}{x+x^2y^2+2z}$$

3) $x^2+y^2+z^2=3$ denklemi ile tanımlanan küre yüzeyinin $A(1,1,1)$ noktasındaki teğet düzleminin denklemini yazınız. (15 puan)

$\vec{\nabla}f(x,y,z)=\langle 2x,2y,2z\rangle \Rightarrow \vec{\nabla}f(1,1,1)=\langle 2,2,2\rangle$, $D_T(x,y,z)=1$ olan

$\forall N(x,y,z)$ için $\vec{\nabla}f(1,1,1)\perp \vec{A}$

$\Rightarrow \vec{\nabla}f(1,1,1)\cdot\vec{AN}=0 \Rightarrow D_T: 2(x-1)+2(y-1)+2(z-1)=0 \Rightarrow 2x+2y+2z=6 \Rightarrow x+y+z=3$



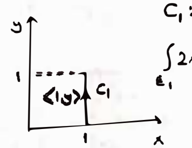
4) $x^2+y^2=1$ çemberi üzerinde $f(x,y)=x^2+2y^2$ fonksiyonunun ekstremumlarını bularak değerlerini hesaplayınız. (20 puan)

$g(x,y)=x^2+y^2=1$

$$\vec{\nabla}f(x,y)=\lambda\vec{\nabla}g(x,y) \Rightarrow \left\{\begin{array}{l} 2x=2x\lambda \Rightarrow \begin{cases} x=0 \Rightarrow y=\pm 1 \\ x\neq 0 \Rightarrow \lambda=1 \end{cases} \\ 4y=2y\lambda \Rightarrow \lambda=1 \Rightarrow y=0 \Rightarrow x=\pm 1 \\ \qquad x^2+y^2=1 \end{array}\right\} \Rightarrow \begin{array}{l} \left.\begin{array}{l}A(0,1)\\B(0,-1)\end{array}\right\} f(0,1)=f(0,-1)=2\ max \\ \\ \left.\begin{array}{l}C(1,0)\\D(-1,0)\end{array}\right\} \Rightarrow f(1,0)=f(-1)=1\ min \end{array}$$

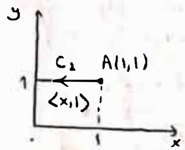
II. yol:

$$\oint 2xy^2dx+6yxdy=\int_{C_1} 2xy^2dx+6yxdy+\int_{C_2} 2xy^2dx+6yxdy+\int_{C_3} 2xy^2dx+6yxdy$$



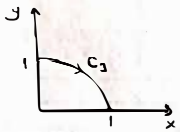
$C_1: \vec{r}_1(y)=\langle 1,y\rangle,\ 0\leq y\leq 1 \Rightarrow x=1 \Rightarrow dx=0;\ dy=dy$

$$\int_{C_1} 2xy^2dx+6yxdy=\int_0^1 6y\,dy=3y^2\Big|_0^1=3$$



$C_2: \vec{r}_2(x)=\langle x,1\rangle,\ 1\leq x\leq 0 \Rightarrow dx=dy;\ y=1\Rightarrow dy=0$

$$\int_{C_2} 2xy^2dx+6yxdy=\int_1^0 2x\,dx=x^2\Big|_1^0=-1$$

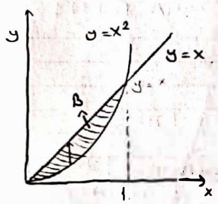


$C_3: \vec{r}_3(\theta)=\langle\cos\theta,\sin\theta\rangle,\ \frac{\pi}{2}\leq\theta\leq 0 \Rightarrow dx=-\sin\theta d\theta;\ dy=\cos\theta d\theta$

$$\int_{C_3} xy^2dx+6yxdy=\int_{\frac{\pi}{2}}^{0}(-2\cos\theta\sin^3\theta+6\cos^2\theta\sin\theta)d\theta=-\frac{3}{2}$$

$$\oint_C 2xy^2dx+6yxdy=3-1-\frac{3}{2}=\frac{1}{2}$$

5) $B=\{(x,y)\mid 0\leq x\leq 1,\ x^2\leq y\leq x\}$ olmak üzere, bölgeyi xy-düzleminde çizerek; $\iint_B (xy+1)dA=?$ (15 puan)

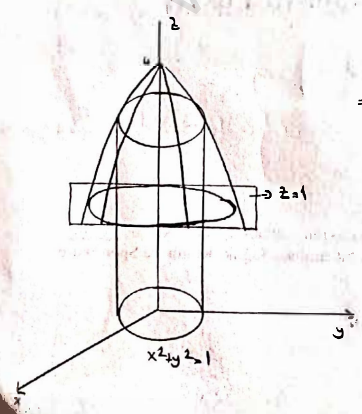


$$\iint_B (xy+1)dA=\int_0^1\int_{x^2}^{x}(xy+1)dx\,dy=\int_0^1\left(x\frac{y^2}{2}+y\right)\Big|_{x^2}^{x}dx$$

$$=\int_0^1\left(x\frac{x^2}{2}+x-x\frac{x^4}{2}-x^2\right)dx=\int_0^1\left(\frac{x^3}{2}+x-\frac{x^5}{2}-x^2\right)dx$$

$$=\frac{x^4}{8}+\frac{x^2}{2}-\frac{x^6}{12}-\frac{x^3}{3}\Big|_0^1=\frac{1}{8}+\frac{1}{2}-\frac{1}{3}-\frac{1}{12}=\frac{5}{24}$$

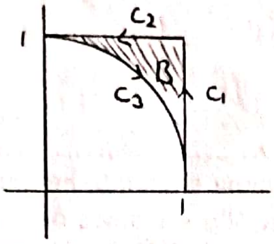
6) $x^2+y^2=1$ silindirinin içinde kalan, $z=4-x^2-y^2$ paraboloidi ve $z=1$ düzlemi ile sınırlı cismin hacmini katlı integral ile hesaplayınız. (15 puan)



$$H=\iiint_B dH=\int_0^{2\pi}\int_0^1(4-r^2-1)r\,dr\,d\theta=\int_0^{2\pi}\int_0^1(3r-r^3)dr\,d\theta$$

$$=2\pi\left(\frac{3r^2}{2}-\frac{r^4}{4}\right)\Big|_0^1=\frac{5\pi}{2}\ br^3$$

7) C_3 eğrisi, orijin merkezli 1. bölgede yarıçapı 1 birim olan çember yayı, C_1, C_2 sırasıyla $x=1$, $y=1$ doğru parçaları ile verilen; $C = C_1 \cup C_2 \cup C_3$ olmak üzere; $\oint 2xy^2dx + 6xy\,dy = ?$ (15 puan)



$$\oint_C 2xy^2dx + 6xy\,dy = \iint_B (6y - 4xy)\,dA = \int_0^1 \int_{\sqrt{1-x^2}}^1 2y(3-2x)\,dy\,dx$$

$$= \int_0^1 (3-2x)\left(y^2\Big|_{\sqrt{1-x^2}}^1\right) dx = \int_0^1 (3-2x)(1-1+x^2)\,dx = \int_0^1 (3-2x)x^2\,dx$$

$$= \int_0^1 (3x^2 - 2x^3)\,dx = \left(x^3 - \frac{1}{2}x^4\right)\Big|_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$$