1) $\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \overbrace{\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}}^{A}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \overbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}^{B} e(t) \qquad e(t) = U(t) = \begin{cases}1 & t \geq 0\\ 0 & t < 0\end{cases} \qquad \begin{bmatrix}X_1(0)\\ X_2(0)\end{bmatrix} = \overbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}^{X(0)} \qquad \mathcal{L}\{U(t)\} = \frac{1}{s}$

Verilen durum denklemini s-bölgesinde çözünüz.

$$X(s) = [sI - A]^{-1} X(0) + [sI - A]^{-1} B\, U(s)$$

Durum geçiş matrisi: $\phi(s) = [sI - A]^{-1} = \left\{\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\right\}^{-1} = \begin{bmatrix}s & -1\\ 2 & s+3\end{bmatrix}^{-1}$ $\quad \Delta = s^2 - 3s + 2$ (-2, -1) $= (s+1)(s+2)$

$$\begin{bmatrix}\frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)}\\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)}\end{bmatrix} = \phi(s)$$

$$X(s) = \underbrace{\begin{bmatrix}\frac{s+3}{\Delta} & \frac{1}{\Delta}\\ -\frac{2}{\Delta} & \frac{s}{\Delta}\end{bmatrix}}_{[sI-A]^{-1}} \underbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}_{X(0)} + \underbrace{\begin{bmatrix}\frac{s+3}{\Delta} & \frac{1}{\Delta}\\ \frac{-2}{\Delta} & \frac{s}{\Delta}\end{bmatrix}}_{[sI-A]^{-1}} \underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{B} \cdot \underbrace{\left[\frac{1}{s}\right]}_{U(s)} = \overbrace{\begin{bmatrix}\frac{s+3}{\Delta}\\ \frac{-2}{\Delta}\end{bmatrix}}^{X_{öz}(s)} + \overbrace{\begin{bmatrix}\frac{1}{\Delta s}\\ \frac{1}{\Delta}\end{bmatrix}}^{X_{zor}(s)}$$

$\frac{s+3}{(s+1)(s+2)} = \frac{a_1}{s+1} + \frac{a_2}{s+2}$ (kökü -1, kökü -2) $\quad -1$ için $\frac{s+3}{s+2} = 2 = a_1$, $\quad -2$ için $\frac{s+3}{s+1} = \frac{1}{-1} = -1 = a_2$

$$= \frac{2}{s+1} - \frac{1}{s+2}$$

$\frac{-2}{(s+1)(s+2)} = \frac{b_1}{s+1} + \frac{b_2}{s+2} \quad -1$ için $\frac{-2}{s+2} = -2 = b_1$, $\quad -2$ için $\frac{-2}{s+1} = 2 = b_2$

$$= \frac{-2}{s+1} + \frac{2}{s+2}$$

$\frac{1}{(s+1)(s+2).s} = \frac{c_1}{s} + \frac{c_2}{s+1} + \frac{c_3}{s+2} \quad 0$ için $\frac{1}{(s+1).(s+2)} = \frac{1}{2} = c_1$, $\quad -1$ için $\frac{1}{(s+2).s} = -1 = c_2$

-2 için $\frac{1}{(s+1).s} = \frac{1}{2} = c_3 \qquad = \frac{1}{2s} - \frac{1}{s+1} + \frac{1}{2(s+2)}$

$\frac{1}{(s+1)(s+2)} = \frac{d_1}{s+1} + \frac{d_2}{s+2} \quad -1$ için $\frac{1}{s+2} = 1 = d_1$, $\quad -2$ için $\frac{1}{s+1} = -1 = d_2$

$$= \frac{1}{s+1} - \frac{1}{s+2}$$

$$\underbrace{\begin{bmatrix} \frac{s+3}{\Delta} \\ \frac{-2}{\Delta} \end{bmatrix}}_{X_{öz}(s)} + \underbrace{\begin{bmatrix} \frac{1}{\Delta s} \\ \frac{1}{\Delta} \end{bmatrix}}_{X_{zor}(s)} \Rightarrow \mathcal{L}^{-1}\{\underline{X}(s)\}$$

sürekli durum bileseni

gecici durum bileseni

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}}_{X_{öz}} + \underbrace{\begin{bmatrix} \frac{1}{2}u(t) - e^{-t} + \frac{1}{2}e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}}_{X_{zor}}$$

2) $\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \overbrace{\begin{bmatrix}1 & 0\\ 1 & -3\end{bmatrix}}^{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \overbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}^{B} e(t)$ $\quad e(t) = \overbrace{\delta(t)}^{\text{birim vurus}}$, $\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$ $\quad y(t) = \overbrace{\begin{bmatrix}-\frac{1}{4} & 1\end{bmatrix}}^{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

s bölgesinde durum denklemlerini çözünüz. Matrissel geçiş (transfer) fonk. bulunuz.

$$\underline{X}(s) = \overbrace{\underbrace{[sI-A]^{-1}}_{\phi(s)}\underline{X}(0)}^{\text{özel çözüm}} + \underbrace{[sI-A]^{-1}B.\overbrace{E(s)}^{U(t)}}_{\text{Zorlanmış çözüm}}$$

$$H(s) = \frac{Y(s)}{U(s)}\Bigg|_{X(0)=0} = C.[sI-A]^{-1}.B$$

↳ Matrissel transfer fonksiyonu (Geçiş işlevi)

$$\phi(s) = [sI-A]^{-1} = \left\{\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - \begin{bmatrix}1 & 0\\ 1 & -3\end{bmatrix}\right\}^{-1} \qquad \phi(s) = \begin{bmatrix}s-1 & 0\\ -1 & s-3\end{bmatrix}^{-1}$$

$$\Delta = s^2 - 2s - 3 = (s-1).(s+3) = 0$$

3 −1

$\boxed{\begin{matrix}s_1 = 1\\ s_2 = -3\end{matrix}}$ Kararsız

$$\begin{bmatrix}\frac{s+3}{(s-1).(s+3)} & 0\\ \frac{1}{(s-1)(s+3)} & \frac{s-1}{(s-1)(s+3)}\end{bmatrix} = \phi(s)$$

$$\begin{bmatrix}X_1\\ X_2\end{bmatrix} = \underbrace{\begin{bmatrix}\frac{1}{s-1} & 0\\ \frac{1}{(s-1)(s+3)} & \frac{1}{s+3}\end{bmatrix}}_{[sI-A]^{-1}}\underbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}_{B}\underbrace{[1]}_{E(s)}$$

$\delta = e(t) = E(s) = 1$

$$\begin{bmatrix}X_1(s)\\ X_2(s)\end{bmatrix} = \begin{bmatrix}\frac{1}{s-1}\\ \frac{1}{(s-1)(s+3)}\end{bmatrix}$$

$$\frac{a_1}{s-1} + \frac{a_2}{s+3}$$

$$a_1 = \frac{1}{4} \qquad a_2 = -\frac{1}{4}$$

$$= \begin{bmatrix}\frac{1}{s-1}\\ \frac{1}{4(s-1)} - \frac{1}{4(s+3)}\end{bmatrix} = \begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} = \begin{bmatrix}e^{t}\\ \frac{1}{4}e^{t} - \frac{1}{4}e^{-3t}\end{bmatrix}$$

↳ sürekli durum bileşeni

↳ Geçici durum bileşeni

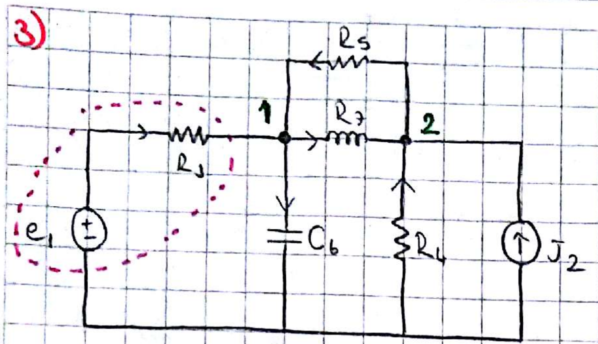
$t \to \infty$ $x_1 \to \infty$, $x_2 \to \infty$ öyleyse kararsızdır.

$$H(s) = \left.\frac{Y(s)}{U(s)}\right|_{X(0)=0} = C.\Phi(s).B \quad \begin{bmatrix} -\frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)(s+3)} & \frac{1}{(s+3)} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \underset{1\times 2}{\begin{bmatrix} -\frac{1}{4} & 1 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} \frac{1}{s-1} \\ \frac{1}{(s-1)(s+3)} \end{bmatrix}} = \left[-\frac{1}{\underset{(s+3)}{4(s-1)}} + \frac{1}{\underset{4}{(s-1)(s+3)}} \right] = \frac{-(s+3)+4}{4(s-1).(s+3)} = \frac{-s-3+4}{4(s-1)(s+3)} = \frac{-\cancel{(s-1)}}{4\cancel{(s-1)}(s+3)}$$

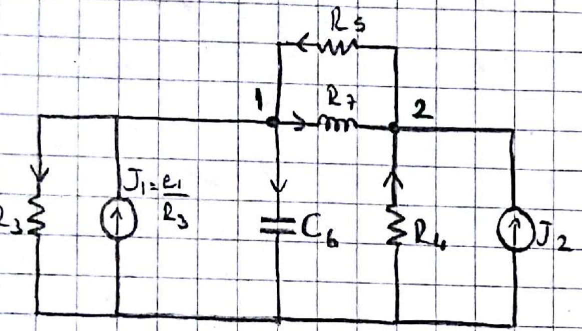
$$= \frac{-1}{4(s+3)} = H(s)$$

3)



Verilen devrenin çözümünü sadece 1 ve 2 denklemlerini kullanarak düğüm denklemleri ile yazınız.

$$X = \begin{bmatrix} V_{d_1} \\ V_{d_2} \end{bmatrix}$$



1) 1) $-J_1 + I_3 + I_6 - I_5 + I_7 = 0$

2) $I_5 - I_7 - I_4 - J_2 = 0$

$V_3(s) = V_{d_1}(s)$

$V_4(s) = -V_{d_2}(s)$

$V_5(s) = V_{d_2}(s) - V_{d_1}(s)$

$V_6(s) = V_{d_1}(s)$

$V_7(s) = V_{d_1}(s) - V_{d_2}(s)$

2) 1) $-\dfrac{E_1(s)}{R_3} + G_3 V_3(s) + sC_6 V_6(s) - C_6 V_6(t_0) - G_5 V_5(s) + \dfrac{V_7(s)}{sL_7} + \dfrac{I_7(t_0)}{s} = 0$

2) $G_5 V_5(s) - \dfrac{V_7(s)}{sL_7} - \dfrac{I_7(t_0)}{s} - G_4 V_4(s) - J_2(s) = 0$

3) 1) $-\dfrac{E_1(s)}{R_3} + G_3 V_{d_1}(s) + sC_6 V_{d_1}(s) - C_6 V_6(t_0) - G_5 (V_{d_2}(s) - V_{d_1}(s)) + \dfrac{V_{d_1}(s) - V_{d_2}(s)}{sL_7} + \dfrac{I_7(t_0)}{s} = 0$

2) $G_5 (V_{d_2}(s) - V_{d_1}(s)) - \dfrac{(V_{d_1}(s) - V_{d_2}(s))}{sL_7} - \dfrac{I_7(t_0)}{s} + G_4 V_{d_2}(s) - J_2(s) = 0$

1) $V_{d_1}(s)\left(G_3 + sC_6 + G_5 + \dfrac{1}{sL_7}\right) + V_{d_2}(s)\left(-G_5 - \dfrac{1}{sL_7}\right) = \dfrac{E_1(s)}{R_3} + C_6 V_6(t_0) + \dfrac{I_7(t_0)}{s}$

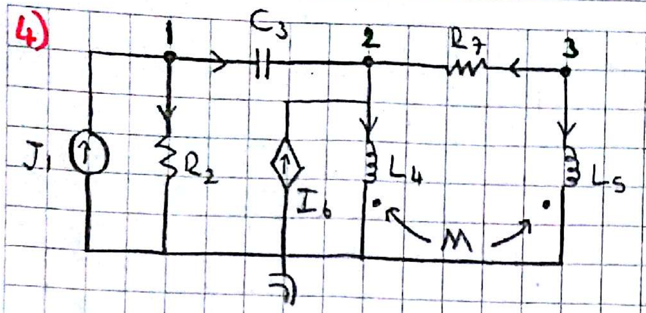
2) $V_{d_1}(s)\left(-G_5 - \dfrac{1}{sL_7}\right) + V_{d_2}(s)\left(G_5 + \dfrac{1}{sL_7} + G_4\right) = \dfrac{I_7(t_0)}{s} + J_2(s)$

4)

$$\begin{array}{c c}
 & \begin{array}{cc} V_{d_1} & \qquad\qquad\qquad V_{d_2} \end{array} \\
\begin{array}{c} \mathrm{I} \\ \\ \mathrm{II} \end{array} & \underbrace{\begin{bmatrix} G_3 + sC_6 + G_5 + \dfrac{1}{sL_7} & -G_5 - \dfrac{1}{sL_7} \\ & \\ -G_5 - \dfrac{1}{sL_7} & G_5 + \dfrac{1}{sL_7} + G_4 \end{bmatrix}}
\end{array}
\begin{bmatrix} V_{d_1} \\ \\ V_{d_2} \end{bmatrix} = \begin{bmatrix} \dfrac{E_1(s)}{R_3} \\ \\ J_2(s) \end{bmatrix} + \begin{bmatrix} C_6 V_6(t_0) + \dfrac{I_7(t_0)}{s} \\ \\ \dfrac{I_7(t_0)}{s} \end{bmatrix}$$

Y_d : Düğümlere admitans matrisi V_d = J_s + Başlangıç değerleri

4)



$$X = \begin{bmatrix} V_{d_1} \\ V_{d_2} \\ V_{d_3} \end{bmatrix} \qquad I_6 = g.V_3$$

$$\begin{bmatrix} V_4(s) \\ V_5(s) \end{bmatrix} = s \begin{bmatrix} L_4 & M \\ M & L_5 \end{bmatrix} \begin{bmatrix} I_4(s) \\ I_5(s) \end{bmatrix} - \begin{bmatrix} L_4 & M \\ M & L_5 \end{bmatrix} \begin{bmatrix} I_4(t_0) \\ I_5(t_0) \end{bmatrix} \qquad \begin{bmatrix} I_4(s) \\ I_5(s) \end{bmatrix} = \begin{bmatrix} L_5/\Delta_s & -M/\Delta_s \\ -M/\Delta_s & L_4/\Delta_s \end{bmatrix} + \begin{bmatrix} I_4(t_0)/s \\ I_5(t_0)/s \end{bmatrix}$$

$$I_4(s) = \frac{L_5}{\Delta_s} V_4(s) - \frac{M}{\Delta_s} V_5(s) + \frac{I_4(t_0)}{s} \qquad I_5(s) = -\frac{M}{\Delta_s} V_4(s) + \frac{L_4}{\Delta_s} V_5(s) + \frac{I_5(t_0)}{s}$$

1) 1) $-I_1 + I_2 + I_3 = 0$

2) $-I_3 + I_4 - I_6 - I_7 = 0$

3) $I_5 + I_7 = 0$

2) 1) $-J_1(s) + G_2 V_2(s) + sC_3 V_3(s) - C_3 V_3(t_0) = 0$

2) $-sC_3 V_3 + C_3 V_3(t_0) + \frac{L_5}{\Delta_s} V_4(s) - \frac{M}{\Delta_s} V_5(s) + \frac{I_4(t_0)}{s} - gV_3 - G_7 V_7 = 0$

3) $-\frac{M}{\Delta_s} V_4(s) + \frac{L_4}{\Delta_s} V_5(s) + \frac{I_5(t_0)}{s} + G_7 V_7(s) = 0$

$V_2 = V_{d_1}(s)$, $V_3 = V_{d_1}(s) - V_{d_2}(s)$, $V_4 = V_{d_2}(s)$, $V_5 = V_{d_3}(s)$, $V_7 = V_{d_3}(s) - V_{d_2}(s)$

3) 1) $-J_1(s) + G_2 V_{d_1}(s) + sC_3 \big(V_{d_1}(s) - V_{d_2}(s)\big) - C_3 V_3(t_0) = 0$

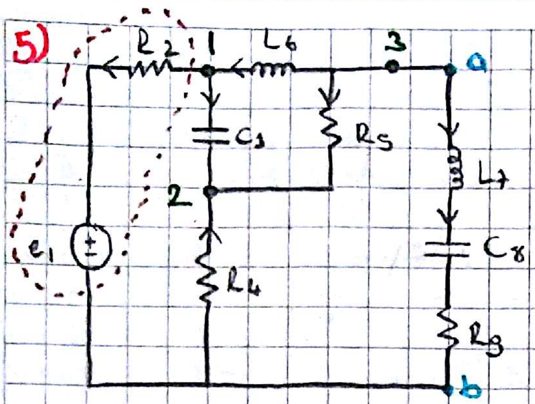
2) $-sC_3\big(V_{d_1}(s) - V_{d_2}(s)\big) + C_3 V_3(t_0) + \frac{L_5}{\Delta_s} V_{d_2}(s) - \frac{M}{\Delta_s} V_{d_3}(s) + \frac{I_4(t_0)}{s} - g\big(V_{d_1}(s) - V_{d_2}(s)\big)$
$- G_7\big(V_{d_3}(s) - V_{d_2}(s)\big) = 0$

3) $-\frac{M}{\Delta_s} V_{d_2}(s) + \frac{L_4}{\Delta_s} V_{d_3}(s) + \frac{I_5(t_0)}{s} + G_7\big(V_{d_3}(s) - V_{d_2}(s)\big) = 0$

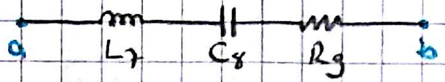
4)

$$\begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array} \begin{array}{c} \begin{array}{ccc} V_{d_1} & V_{d_2} & V_{d_3} \end{array} \\ \underbrace{\begin{bmatrix} G_2 + sC_3 & -sC_3 & 0 \\ -sC_3 - g & sC_3 + \frac{L_5}{\Delta_s} + g + G_7 & -\frac{M}{\Delta_s} - G_7 \\ 0 & -\frac{M}{\Delta_s} - G_7 & \frac{L_4}{\Delta_s} + G_7 \end{bmatrix}}_{Y_d \text{ (Admitans matrisi)}} \end{array} \underbrace{\begin{bmatrix} V_{d_1}(s) \\ V_{d_2}(s) \\ V_{d_3}(s) \end{bmatrix}}_{V_d(s)} = \underbrace{\begin{bmatrix} J_1(s) \\ 0 \\ 0 \end{bmatrix}}_{J(s)} + \underbrace{\begin{bmatrix} C_3 V_3(t_0) \\ -C_3 V_3(t_0) - \frac{I_4}{s} \\ -\frac{I_5}{s} \end{bmatrix}}_{\text{Başlangıç değerleri}}$$

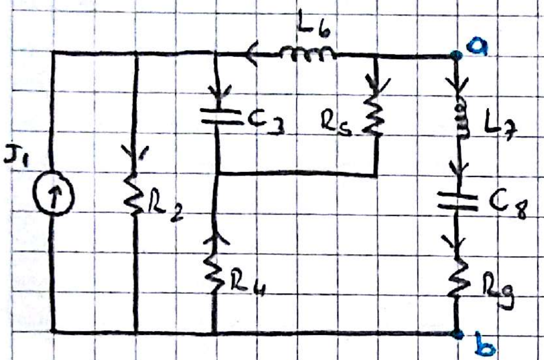
5)

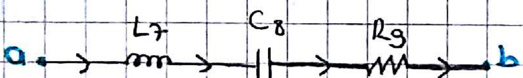


Başlangıç değerleri sıfırdan farklı. 1, 2, 3 düğüm denklemlerini doğrudan yazınız.



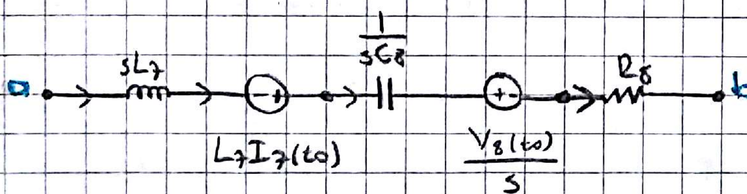
a, b düğümünü ortadan kaldırmak için bu elemanların akım eşdeğer empedansını bulup çizeriz. (Akım kaynağı olduğu için akım eşdeğeri). Elemanlar seri olduğu için öncelikle gerilim eşdeğeri çizilir daha sonra akım eşdeğe geçilir.





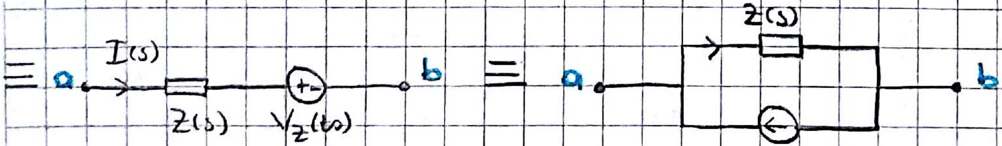
$$I_7(s) = \frac{1}{sL_7} V_7(s) + \frac{I_7(t_0)}{s} \qquad V_7(s) = sL_7 I_7(s) - L_7 I_7(t_0)$$

$$I_8(s) = sC_8 V_8(s) - C_8 V_8(t_0) \qquad V_8(s) = \frac{1}{sC_8} I_8(s) + \frac{V_8(t_0)}{s}$$

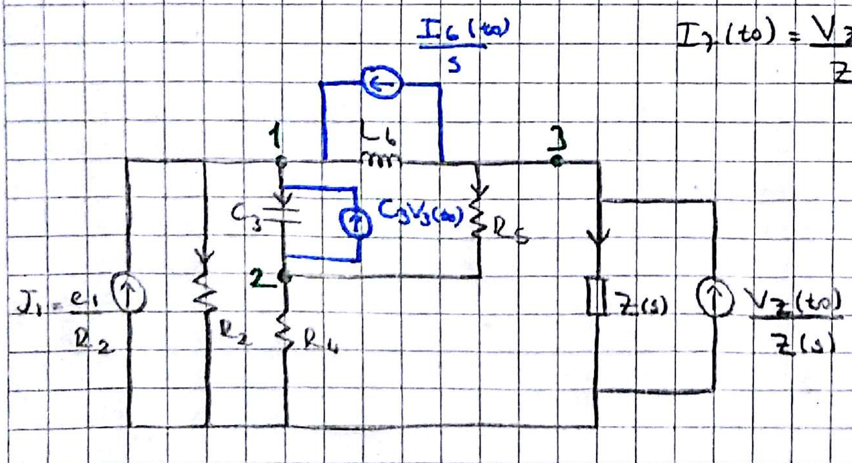


$$Z(s) = R_8 + sL_7 + \frac{1}{sC_8}$$

$$V_Z(t_0) = \frac{V_8(t_0)}{s} - L_7 I_7(t_0)$$



$$I_Z(t_0) = \frac{V_Z(t_0)}{Z(s)}$$



$$\begin{bmatrix} G_2 + sC_3 + \frac{1}{sL_6} & -sC_3 & -sL_6 \\ -sC_3 & sC_3 + G_4 + G_5 & -G_5 \\ -sL_6 & -G_5 & G_5 + \frac{1}{sL_6} + \frac{1}{Z(s)} \end{bmatrix} \begin{bmatrix} Vd_1(s) \\ Vd_2(s) \\ Vd_3(s) \end{bmatrix} = \begin{bmatrix} \frac{E_1}{R_2} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C_3V_3(t_0) + \frac{I_6(t_0)}{s} \\ -C_3V_3(t_0) \\ \frac{-I_6(t_0)}{s} + \frac{V_Z(t_0)}{Z(s)} \end{bmatrix}$$