1) $x^3y'' + 3x^2y' + 2xy = \operatorname{cosec}(\ln x)$

$x^2y'' + 3xy' + 2y = \dfrac{1}{x\sin(\ln x)}$, $x = e^t \Rightarrow \underbrace{(D^2 - D + 3D + 2)}_{(D^2+1)^2+1} y = \dfrac{1}{e^t \cdot \sin t}$

$y_h = e^{-t}(c_1 \sin t + c_2 \cos t)$

$$W = \begin{vmatrix} e^{-t}\sin t & e^{-t}\cos t \\ e^{-t}(-\sin t + \cos t & e^{-t}(-\cos t - \sin t) \end{vmatrix} = -e^{-2t}$$

$$u_1 = \int \frac{-1}{-e^{-2t}} \cdot \frac{1}{e^t \cdot \sin t} \cdot e^{-t}\cos t\, dt = \ln \sin t$$

$$u_1 = \int \frac{1}{-e^{-2t}} \cdot \frac{1}{e^t \sin t} \cdot e^{-t} \sin t\, dt = -t$$

$\Rightarrow y_p = u_1 y_1 + u_2 y_2$

$\Rightarrow y = e^{-t}(c_1 \sin t + c_2 \cos t) + \ln(\sin t) \cdot e^{-t}\sin t - te^{-t}\cos t$

$y = x^{-t}(c_1 \sin(\ln x) + c_2 \cos(\ln x) + \ln(\sin(\ln x))\, x^{-1}\sin(\ln x) - \ln x \cdot x^{-1} \cdot \cos(\ln x)$

2) $(2y^2+2x+1)dx + 2ydy = 0$ (integral çarpanı yardımıyla çöz)

$M_y = 4y$, $N_x = 0$, $\frac{M_y - N_x}{N} = 2 \Rightarrow M(x) = e^{2x}$ integral çarpanı

$e^{2x}(2y^2+2x+1)dx + 2ye^{2x}dy = 0$

$M_y = 4ye^{2x} = N_x \Rightarrow$ TDD

$F_y = 4ye^{2x} \Rightarrow F = y^2e^{2x} + h(x)$

$F_x = 2y^2e^{2x} + h'(x) = 2y^2e^{2x} + 2xe^{2x} + e^{2x}$

$\Rightarrow h(x) = \frac{1}{2}e^{2x}(2x+1) - \frac{1}{2}e^{2x} + C$

$\Rightarrow F(x,y) = e^{2x}(y^2+x) = C$

3) Aşağıdaki Laplace / ters Laplace dönüşümlerini hesaplayınız.

a)

$$\mathcal{L}^{-1}\left[\frac{2s-3}{s^2-6s+13}\right] = ? \qquad \mathcal{L}^{-1}\left[\frac{2(s-3)+3}{(s-3)^2+4}\right] = 2e^{3t}\cos 2t + \frac{3}{2}e^{3t}\sin 2t$$

b) $\mathcal{L}[(\sin t)(\cos 3t)] = ?$ $\quad \sin t\cos 3t = \frac{1}{2}\sin 4t - \frac{1}{2}\sin 2t$

$$\mathcal{L}[\sin t \cdot \cos 3t] = \frac{1}{2}\cdot\frac{4}{s^2+16} - \frac{1}{2}\cdot\frac{2}{s^2+4}$$

4) $(x+1)^2 y'' - 3(x+1)y' + 3y = 0$ denkleminin bir özel çözümü $y_1(x) = x+1$ olduğuna göre genel çözümünü bulunuz.

$y = (x+1)v$, $y' = v + (x+1)v'$, $y'' = 2v' + (x+1)v''$

$\Rightarrow (x+1)^2 [2v' + (x+1)v''] - 3(x+1)[v + (x+1)v'] + 3(x+1)v = 0$

$(x+1)v'' = v'$, $v' = V \Rightarrow v'' = V \Rightarrow \frac{dV}{V} = \frac{dx}{x+1} \Rightarrow V = c_1(x+1) = v'$

$\Rightarrow v = c_1\left(\frac{x^2}{2} + x\right) + c_2 = \frac{y}{(x+1)}$

$\Rightarrow y = c_1(x+1)\left(\frac{x^2}{2} + x\right) + c_2(x+1)$

5) $ty''-(t-1)y'-y=-3$, $y(0)=-2$, $y'(0)=1$ (Laplace dönüşümü ile)

$-(-y'(0)-sy(0)+s^2F)'+(-y(0)+sF)'-y(0)+sF-F=-\frac{3}{s}$

$-2sF-s^2F'+F+sF'+sF-F=-\frac{3}{s}$

$\Rightarrow F'+\frac{1}{s-1}F=\frac{3}{s^2(s-1)}$

$M(s)=s-1 \Rightarrow (s-1)F=-\frac{3}{s}+c_1$

$\Rightarrow F(s)=\frac{-3+c_1s}{s(s+1)}=\frac{A}{s}+\frac{B}{s-1}$

$\Rightarrow -3+c_1s=A(s-1)+Bs \Rightarrow A=3,\ B=c_1-3$

$\Rightarrow y(t)=3+(c_1-3)e^t$

$y(0)=3+(c_1-3)=-2 \Rightarrow c_1=-2$

6) $\left.\begin{aligned} x'+2y'-x+2y &= \cos t \\ 2x'+y'+4x-3y &= 2\sin t \end{aligned}\right\}$ diferansiyel denklem sisteminin genel çözümünü bul.

D-D / $(D-1)x+(2D+2)y=\cos t$ $\quad\Rightarrow (D^2-4D+3)x+(D-3)(2D+2)y=-\sin t-3\cos t$

2D+2 / $(2D+4)x+(D-3)y=2\sin t$ $\quad\Rightarrow (4D^2+2D+6)x+(D-3)(2D+2)y=4\sin t+4\cos t$

$(3D^2+16D+5)x=5\sin t+7\cos t$

$\begin{matrix}1 & & 5\\ & \times & \\ 3 & & 1\end{matrix} \Rightarrow x_h=c_1e^{-5t}+c_2e^{-\frac{1}{3}t}$

$x_p=a_1\sin t+a_2\cos t$

$x_p'=a_1\cos t-a_2\sin t$

$x_p''=-a_1\sin t-a_2\cos t$

$$\Rightarrow\begin{cases}-3a_1-16a_2+5a_1=5\\ -3a_2+16a_1+5a_2=7\end{cases}\Rightarrow\begin{matrix}2a_1-16a_2=5\\ 16a_1+2a_2=7\end{matrix}\Rightarrow 130a_1=61\Rightarrow a_1=\frac{61}{130}$$

$$a_2=-\frac{33}{130}$$

$$\Rightarrow x(t)=c_1e^{-5t}+c_2e^{-\frac{1}{3}t}+\frac{61}{130}\sin t-\frac{33}{130}\cos t$$

$$2y'+2y=\cos t+c_1e^{-5t}+c_2e^{-\frac{1}{3}t}+\frac{61}{130}\sin t-\frac{33}{130}\cos t+5c_1e^{-5t}+\frac{c_2}{3}e^{-\frac{1}{3}t}-\frac{61}{130}\cos t$$

$$\hookrightarrow -\frac{33}{130}\sin t$$

$$2(D+1)y=\frac{36}{130}\cos t+\frac{28}{130}\sin t+6c_1e^{-5t}+\frac{4}{3}c_2e^{-\frac{1}{3}t}$$

$y_h=c_3e^{-t}$

$y_p=a_1\cos t+a_2\sin t+a_3e^{-5t}+a_4e^{-\frac{1}{3}t}$

$y_p'=a_1\sin t+a_2\cos t-5a_3e^{-5t}-\frac{1}{3}a_4e^{-\frac{1}{3}t}$

$$\left.\begin{aligned} a_1+a_2&=\frac{18}{130}\\ a_2-a_1&=\frac{14}{130}\end{aligned}\right\}\quad a_2=\frac{16}{130},\quad a_1=\frac{2}{130}$$

$-4a_3=3c_1\Rightarrow a_3=-\frac{3}{4}c_1$

$\frac{2}{3}a_4=\frac{2}{3}c_2\Rightarrow a_4=c_2$

$$y = C_3 e^{-t} + \frac{2}{130}\cos t + \frac{16}{130}\sin t - \frac{3}{4} C_1 e^{-5t} + C_2 e^{-\frac{1}{3}t}$$

2. Denklem

$$-10C_1 e^{-t} + \frac{2}{3} C_2 e^{-\frac{1}{3}t} + \frac{122}{130}\cos t + \frac{66}{130}\sin t - C_3 e^{-t} - \frac{2}{130}\cos t + \frac{16}{130}\sin t$$

$$+ \frac{15}{4} C_1 e^{-5t} - \frac{1}{3} C_2 e^{-\frac{1}{3}t} + 4C_1 e^{-5t} + 4C_2 e^{-\frac{1}{3}t} + \frac{244}{130}\sin t - \frac{132}{130}\cos t$$

$$-3C_3 e^{-t} - \frac{6}{130}\cos t - \frac{48}{130}\sin t + \frac{9}{4} C_1 e^{-5t} - 3C_2 e^{-\frac{1}{3}t} = 2\sin t$$

$\Rightarrow -4C_3 = 0 \quad C_3 = 0$