Diferansiyel denklemler Final

1) $x^3y''+3x^2y'+2xy=\csc(\ln x)$

$x^2y''+3xy'+2y=\dfrac{1}{x\sin(\ln x)}$ $\quad x=e^t \Rightarrow (D^2-D+3D+2)y=\dfrac{1}{e^t.\sin t}$

$y_h=e^{-t}(c_1\sin t+c_2\cos t)$

$W=\begin{vmatrix} e^{-t}\sin t & e^{-t}\cos t \\ e^{-t}(-\sin t+\cos t) & e^{-t}(-\cos t-\sin t)\end{vmatrix}=-e^{-2t}$

$u_1=\int\dfrac{-1}{-e^{-2t}}\cdot\dfrac{1}{e^t.\sin t}\,e^{-t}.\cos t\,dt=\ln\sin t$

$u_2=\int\dfrac{1}{-e^{-2t}}\cdot\dfrac{1}{e^t.\sin t}.e^{-t}\sin t\,dt=-t$

$\Rightarrow y_p=u_1y_1+u_2y_2$

$\Rightarrow y=e^{-t}(c_1\sin t+c_2\cos t)+\ln(\sin t).e^{-t}.\sin t-t.e^{-t}\cos t$

$y=x^{-t}(c_1\sin(\ln x)+c_2\cos(\ln x)+\ln(\sin(\ln x))\,x^{-1}\sin(\ln x)-\ln x.x^{-1}.\cos(\ln x)$

2) $(2y^2+2x+1)dx+2y\,dy=0$ (integral çarpanı yardımıyla çözün)

$M_y=4y$, $N_x=0$ $\quad \dfrac{M_y-N_x}{N}=2 \Rightarrow \mu(x)=e^{2x}$: integral çarpanı

$e^{2x}(2y^2+2x+1)dx+2ye^{2x}dy=0$
$M_y=4ye^{2x}=N_x \Rightarrow$ TDD

$F_y=4ye^{2x}\Rightarrow F=y^2e^{2x}+h(x)$
$F_x=2y^2e^{2x}+h'(x)=2y^2e^{2x}+2xe^{2x}+e^{2x}$
$\Rightarrow h(x)=\frac{1}{2}e^{2x}(2x+1)-\frac{1}{2}e^{2x}+C$

$\Rightarrow F(x,y)=e^{2x}(y^2+x)=c$

3) Aşağıdaki Laplace / ters Laplace dönüşümlerini hesaplayınız.

a) $\mathcal{L}^{-1}\left[\dfrac{2s-3}{s^2-6s+13}\right]=?$ $\quad \mathcal{L}^{-1}\left[\dfrac{2(s-3)+3}{(s-3)^2+4}\right]=2e^{3t}\cos 2t+\frac{3}{2}e^{3t}\sin 2t$

b) $\mathcal{L}[(\sin t)(\cos 3t)]=?$ $\quad \sin t\cos 3t=\frac{1}{2}\sin 4t-\frac{1}{2}\sin 2t \Rightarrow \mathcal{L}[\sin t.\cos 3t]=\frac{1}{2}\cdot\frac{4}{s^2+16}-\frac{1}{2}\cdot\frac{2}{s^2+4}$

c) $\mathcal{L}^{-1}\left[\dfrac{4s-e^{-5}}{(s-2)^3}\right]=?$ $\quad \mathcal{L}^{-1}\left[\dfrac{4(s-2)+3-e^{-5}}{(s-2)^3}\right]=\mathcal{L}^{-1}\left[\dfrac{4}{(s-2)^3}+\dfrac{5}{(s-2)^3}-\dfrac{e^{-5}}{(s-2)^3}\right]$

$=4e^{2t}t+4e^{2t}t^2-u_1(t).\frac{1}{2}e^{2(t-1)}(t-1)^2$

d) $f(t)=\begin{cases}2t & , t<1\\ 1 & , 1\leq t<3\\ \sin t-e^{2t} & , t\geq 3\end{cases} \Rightarrow \mathcal{L}[f(t)]=?$

$f(t)=2t\,u_1(t)+\underbrace{(1-2t)}\,u_2(t)+\underbrace{(\sin t-e^{2t}-1)}\,u_3(t)$

$1-2t=1-2.(t-1+1)$
$=-1-2(t-1)$

$=\sin(t-3+3)-e^{2(t-3+3)}-1$
$=\sin(t-3)\cos 3+\cos(t-3)\sin 3-e^{-3}.e^{2(t-3)}-1$

$\Rightarrow\mathcal{L}[f(t)]=\dfrac{2}{s^2}+e^{-s}\left\{-\dfrac{1}{s}-\dfrac{2}{s^2}\right\}+e^{-3s}\left\{\cos 3\dfrac{1}{s^2+1}+\sin 3\dfrac{s}{s^2+1}-e^{t}\dfrac{1}{s-2}-\dfrac{1}{s}\right\}$

4) $(x+1)^2y''-3(x+1)y'+3y=0$ denkleminin bir özel çözümü $y_1(x)=x+1$ olduğuna göre genel çözümünü bulunuz.

$y=(x+1)u$, $y'=u+(x+1)u'$, $y''=2u'+(x+1)u''$

$\Rightarrow (x+1)^2\left[2u'+(x+1)u''\right]-3(x+1)\left[u+(x+1)u'\right]+3(x+1)u=0$

$(x+1)u''=u'$, $u'=v \Rightarrow u''=v' \Rightarrow \frac{dv}{v}=\frac{dx}{x+1} \Rightarrow v=c_1(x+1)=u'$

$\Rightarrow u=c_1\left(\frac{x^2}{2}+x\right)+c_2=\frac{y}{(x+1)} \Rightarrow y=c_1(x+1)\left(\frac{x^2}{2}+x\right)+c_2(x+1)$

5) $ty''-(t-1)y'-y=-3$, $y(0)=-2$, $y'(0)=1$ (Laplace dönüşümü ile)

$-(-y'(0)-sy(0)+s^2F)'+(-y(0)+sF)'-y(0)+sF-F=\frac{-3}{s}$

$-2sF-s^2F'+F+sF'+sF-F=-\frac{3}{s} \Rightarrow F'+\frac{1}{s-1}F=\frac{3}{s^2(s-1)}$

$\mu(s)=s-1 \Rightarrow (s-1)F=-\frac{3}{s}+c_1$

$\Rightarrow F(s)=\frac{-3+c_1s}{s(s-1)}=\frac{A}{s}+\frac{B}{s-1} \Rightarrow -3+c_1s=A(s-1)+Bs \Rightarrow A=3,\ B=c_1-3$

$\Rightarrow y(t)=3+(c_1-3)e^t$ $\quad y(0)=3+(c_1-3)=-2 \Rightarrow c_1=-2$

6) $\left.\begin{array}{l} x'+2y'-x+2y=\cos t \\ 2x'+y'+4x-3y=2\sin t \end{array}\right\}$ diferansiyel denklem sisteminin genel çözümünü bulunuz.

$\left.\begin{array}{l} D-3/\ (D-1)x+(2D+2)y=\cos t \\ 2D+2/\ (2D+4)x+(D-3)y=2\sin t \end{array}\right\}$ $\quad \begin{array}{l} (D^2-4D+3)x+(D-3)(2D+2)y=-\sin t-3\cos t \\ (4D^2+12D+8)x+(D-3)(2D+2)y=4\sin t+4\cos t \\ \hline (3D^2+16D+5)x=5\sin t+7\cos t \end{array}$

$\begin{array}{cc} 3 & 5 \\ 1 & 1 \end{array} \Rightarrow x_h=c_1e^{-5t}+c_2e^{-\frac{1}{3}t}$

$x_p=a_1\sin t+a_2\cos t$

$x_p'=a_1\cos t-a_2\sin t$

$x_p''=-a_1\sin t-a_2\cos t$

$\Rightarrow \begin{array}{l} -3a_1-16a_2+5a_1=5 \\ -3a_2+16a_1+5a_2=7 \end{array} \Rightarrow \begin{array}{l} 2a_1-16a_2=5 \\ 16a_1+2a_2=7 \end{array} \Rightarrow 130a_1=61 \Rightarrow a_1=\frac{61}{130},\ a_2=-\frac{33}{130}$

$\Rightarrow x(t)=c_1e^{-5t}+c_2e^{-\frac{1}{3}t}+\frac{61}{130}\sin t-\frac{33}{130}\cos t$

$2y'+2y=\cos t+c_1e^{-5t}+c_2e^{-\frac{1}{3}t}+\frac{61}{130}\sin t-\frac{33}{130}\cos t+5c_1e^{-5t}+\frac{c_2}{3}e^{-\frac{1}{3}t}-\frac{61}{130}\cos t-\frac{33}{130}\sin t$

$2(D+1)y=\frac{36}{130}\cos t+\frac{28}{130}\sin t+6c_1e^{-5t}+\frac{4}{3}c_2e^{-\frac{1}{3}t}$

$y_h=c_3e^{-t}$ $\quad \left.\begin{array}{l} y_p=a_1\cos t+a_2\sin t+a_3e^{-5t}+a_4e^{-\frac{1}{3}t} \\ y_p'=a_1\sin t+a_2\cos t-5a_3e^{-5t}-\frac{1}{3}a_4e^{-\frac{1}{3}t} \end{array}\right\}$ $\begin{array}{l} a_1+a_2=\frac{18}{130} \\ a_2-a_1=\frac{14}{130} \end{array}$ $\quad \begin{array}{l} -4a_3=3c_1 \Rightarrow a_3=-\frac{3}{4}c_1 \\ \frac{2}{3}a_4=\frac{2}{3}c_2 \Rightarrow a_4=c_2 \end{array}$

$a_2=\frac{16}{130}$, $a_1=\frac{2}{130}$

$\Rightarrow y=c_3e^{-t}+\frac{2}{130}\cos t+\frac{16}{130}\sin t-\frac{3}{4}c_1e^{-5t}+c_2e^{-\frac{1}{3}t}$

2. denklem: $-10c_1e^{-5t}-\frac{2}{3}c_2e^{-\frac{1}{3}t}+\frac{122}{130}\cos t+\frac{66}{130}\sin t-c_3e^{-t}-\frac{2}{130}\cos t+\frac{16}{130}\sin t+\frac{15}{4}c_1e^{-5t}-\frac{1}{3}c_2e^{-\frac{1}{3}t}$

$+4c_1e^{-5t}+4c_2e^{-\frac{1}{3}t}+\frac{244}{130}\sin t-\frac{132}{130}\cos t-3c_3e^{-t}-\frac{6}{130}\cos t-\frac{48}{130}\sin t+\frac{9}{4}c_1e^{-5t}-3c_2e^{-\frac{1}{3}t}=2\sin t$

$\Rightarrow -4c_3=0 \Rightarrow c_3=0$